PMC
Precision MicroControl Corp.

TechNOTE

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Summary

PMC's MultiFlex family of motion controllers feature versatile trajectory generation algorithms which work in conjunction with fast PID servo algorithms to provide precise control and coordination of multi-axis motion. This tech note provides information about how PMC controllers calculate motion trajectories.

More Information

Closed-loop servo position-mode moves in PMC motion controllers are based on the operation of two entities in the control system; the trajectory generator and the PID loop. The trajectory generator is responsible for generating a velocity profile which provides optimal target positions along this profile at a rate of 1 kHz. The PID algorithm then performs higher-rate position-feedback filter calculations at a rate of 4 kHz to control the command voltage to the motor. Since the trajectory is generated at ¼ the rate of the control loop, the system performs a linear interpolation between optimal trajectory positions, producing a smoothed set of positions to the control loop.

The trajectory generator can calculate three basic types of velocity profiles. The most commonly used velocity profile is the trapezoidal profile, which generates a step-function in acceleration. The trapezoidal profile reaches maximum velocity in the fastest time, but can result in somewhat longer settling times than an s-curve profile. There are two families of s-curve profile. One is based upon a piecewise-continuous sine acceleration curve. The other uses a trapezoidal acceleration profile that is faster to reach full velocity but has somewhat higher jerk - the time derivative of acceleration. The final profile is the parabolic type which generates a monotonically decreasing linear acceleration function and corresponding second-order velocity curve. This is the smoothest and longest time-to-velocity profile.

An exact comparison of the relative performance of these profiles is presented at the end of this document.

A position move can be fundamentally described based on three parameters, maximum Acceleration, maximum Deceleration and maximum Velocity, defined below in terms of encoder position counts as the units of distance;

A	maximum acceleration ($\frac{counts}{\sec^2}$)
D	maximum deceleration ($\frac{counts}{\sec^2}$)
V	maximum velocity ($\frac{counts}{\sec}$)

An additional scaling factor must be considered to understand the normalization that takes place in the controller;

N	trajectory updates per sec. ($\sec^{-1}$)

In the current revision of the controller firmware, the value of N is fixed at 1000. In the following development, the values of A and D are assumed to be the same, although this is not a requirement and the examples presented can be easily extended to cases where they are different.

Two pre-calculated values are required to define the trajectory based upon the parameters described above,

T_A	acceleration time (s)
S_A	acceleration distance (counts)

Once the total move distance, S_{TOT} , is defined, all other characteristics of the trajectory can be derived from these according to,

$S_V = S_{TOT} - 2S_A - S_D$	constant velocity distance (counts)
$T_V = \frac{S_V}{V}$	constant velocity time (s)

Before calculating the expected trajectory values, the user parameters must be scaled to controller units to allow correct interpretation of the updated acceleration, velocity and position values produced by each cycle through the trajectory generator. The scaling can be performed according to the following,

$ACC = A / N^2$ controller acceleration$\left(\frac{counts}{cycle^2}\right)$

$VEL = V / N$ controller velocity $\left(\frac{counts}{cycle}\right)$

The following pages describe the method of determining T_A and S_A for the trapezoidal, s-curve and parabolic profiles in detail. In all cases, the trajectory is defined by an acceleration profile and the velocity and position curves are obtained by successive integration with respect to time.

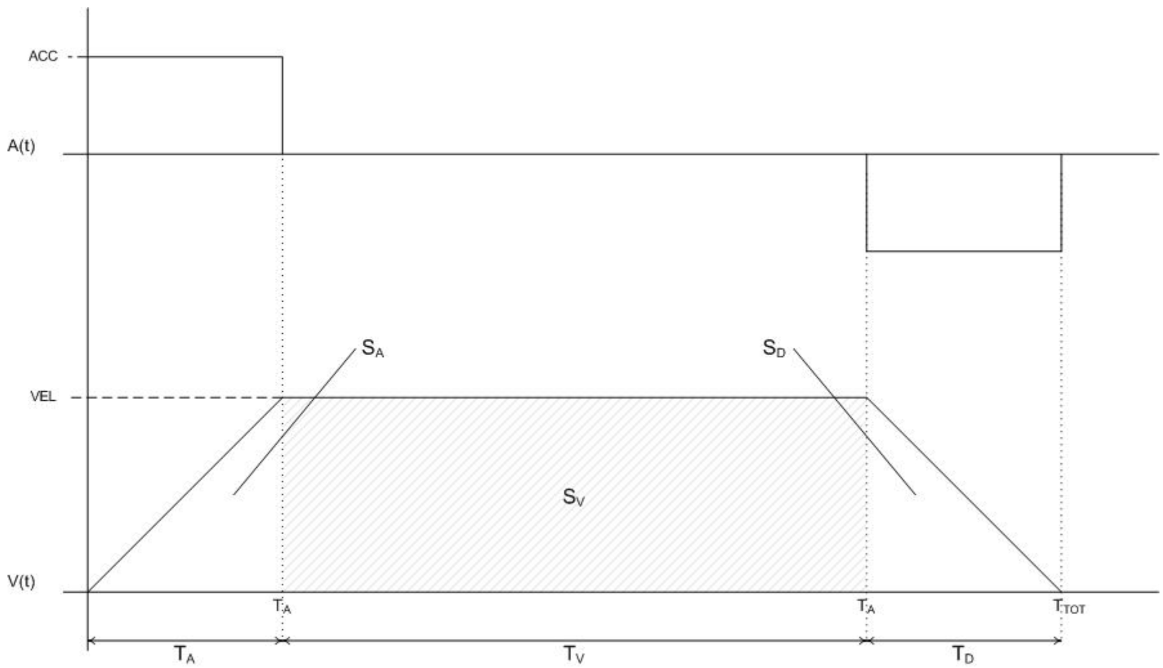
These expressions are developed as continuous time functions involving the variable t. It should be noted that time must be scaled in a similar manner to acceleration and velocity. Since the controller performs discrete calculations at the rate of the trajectory update rate N, the discrete time variable T must be used. The relationship between continuous and discrete controller time variables is

$$t = T / N$$
$$= T / 1000$$

where t has units of seconds and the discrete variable T has units of cycles.

In the following profile descriptions, expressions are given for acceleration, velocity and position in terms of continuous time for the **acceleration phase of the profile only**. Similar expressions can be derived for the constant velocity and deceleration phases of the curve. Finally, the pre-calculated acceleration times and distances – T_A and S_A - are given for each profile type.

Trapezoidal Velocity Profile



$$A(t) = ACC$$

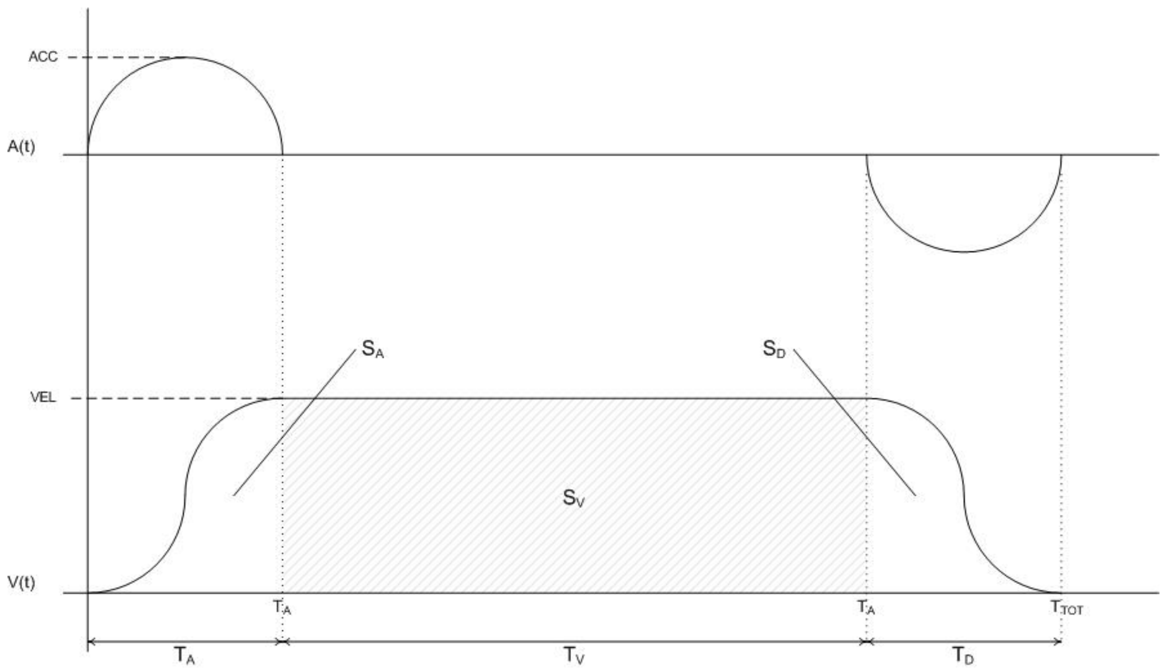
$$\begin{aligned} V(t) &= \int A(t)dt \\ &= \int ACC\ dt \\ &= ACC * (t) \end{aligned}$$

$$\begin{aligned} S(t) &= \int V(t)dt \\ &= \int ACC(t)\ dt \\ &= \frac{1}{2}(ACC) * t^2 \end{aligned}$$

The resulting pre-calculated values for the trapezoidal velocity profile are:

$$\boxed{\begin{aligned} T_A &= \frac{VEL}{ACC} \\ S_A &= \frac{1}{2}(T_A * VEL) \end{aligned}}$$

S-Curve Velocity Profile (sinusoidal)



$$A(t) = ACC\left[\sin \Pi\left(\frac{t}{T_A}\right)\right]$$

$$V(t) = \int A(t)dt = \int ACC \sin 2\Pi t \; dt$$
$$= \left(\frac{VEL}{2}\right)\left(1 - \cos \Pi\left(\frac{t}{T_A}\right)\right)$$

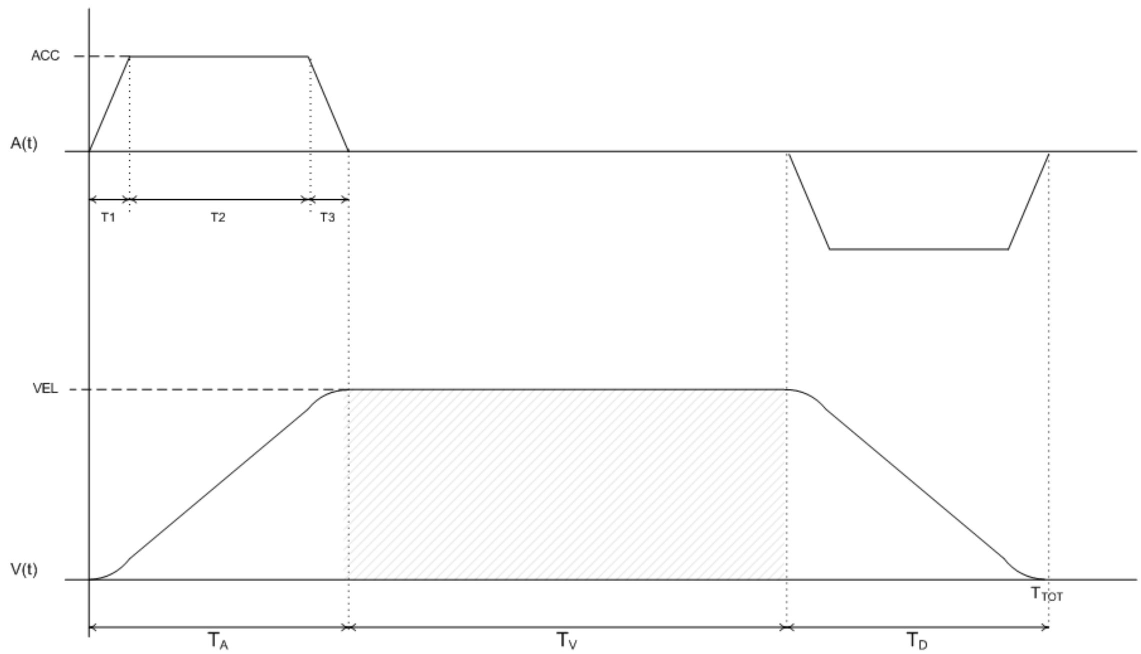
$$S(t) = \int V(t)dt = \int ACC(1 - \cos 2\Pi t)\; dt$$

$$= \left(\frac{VEL * T_A}{2\Pi}\right)\left[\sin \Pi\left(\frac{t}{T_A}\right)\right] + \left(\frac{ACC * T_A}{\Pi} * t\right)$$

The resulting pre-calculated values for the sinusoidal s-curve velocity profile are:

$$T_A = \frac{\Pi}{2}\left(\frac{VEL}{ACC}\right)$$
$$S_A = \frac{1}{2}\left(T_A * VEL\right)$$

S-Curve Velocity Profile (trapezoidal)



$$A(t) = ACC\left[\left(\frac{t}{T_1}\right)\right] + \left[ACC(t)\right] + ACC\left[1-\left(\frac{t}{T_3 - T_2}\right)\right]$$

The velocity and position expressions for the positive acceleration ramp ($t < T_1$) are shown below. Similar expressions can be derived for the remainder of acceleration.

$$V(t) = \int A(t)dt = \int ACC\left[\left(\frac{t}{T_1}\right)\right]dt$$

$$= (ACC * t)\left(\left(\frac{t}{2T_1}\right)\right)$$

$$S(t) = \int V(t)dt = \int (ACC * t)\left(\left(\frac{t}{2T_1}\right)\right)dt$$

$$= ACC * 0.5 * t^2 * \left(\frac{t}{3T_1}\right)$$

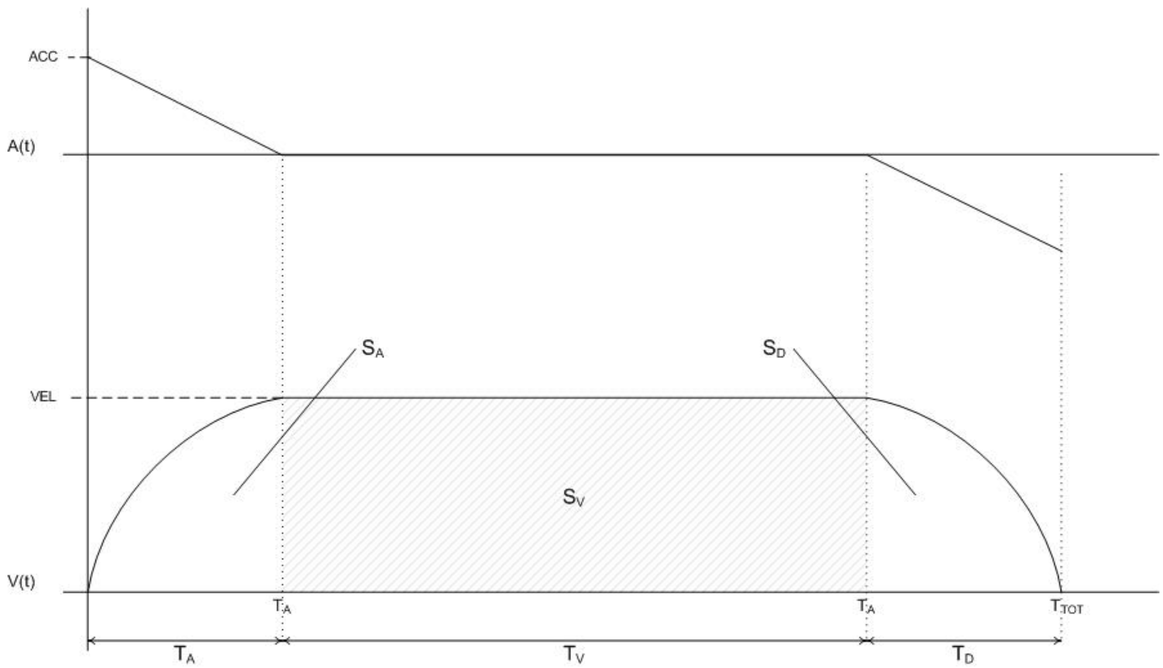
The resulting pre-calculated values for the trapezoidal s-curve velocity profile are:

$$T_A = \frac{1}{RAMP}\left(\frac{VEL}{ACC}\right)$$

$$S_A = \frac{1}{2}(T_A * VEL)$$

Where $RAMP = \frac{T_1}{T_A}$

Parabolic Velocity Profile



$$A(t) = ACC\left[1 - \left(\frac{t}{T_A}\right)\right]$$

$$V(t) = \int A(t)dt = \int ACC\left[1 - \left(\frac{t}{T_A}\right)\right]dt$$

$$= (ACC * t)\left(1 - \left(\frac{t}{2T_A}\right)\right)$$

$$S(t) = \int V(t)dt = \int (ACC * t)\left(1 - \left(\frac{t}{2T_A}\right)\right)dt$$

$$= ACC * 0.5 * t^2 * \left(1 - \frac{t}{3T_A}\right)$$

The resulting pre-calculated values for the parabolic velocity profile are:

$$\boxed{\begin{aligned} T_A &= 2\left(\frac{VEL}{ACC}\right) \\ S_A &= \frac{2}{3}(T_A * VEL) \end{aligned}}$$

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To illustrate the performance of the four types of velocity profile, the example below can be used, based upon the following set of user parameters,

$A = 100{,}000\ \frac{counts}{sec^2}$ $V = 40{,}000\ \frac{counts}{sec}$ $S_{TOT} = 40{,}000$ counts

resulting in the scaled values,

$ACC = 0.1\ \frac{counts}{cycle^2}$ $VEL = 40\ \frac{counts}{cycle}$

The resulting time and position data for the four trajectory profiles are:

parameter	trapezoidal	sinusoidal s-curve	trapezoidal s-curve (0.1)	parabolic
T_A (cycles)	400	628	444	800
S_A (counts)	8000	12,560	8880	10667
S_V (counts)	24000	14,880	22240	18667
T_V (cycles)	600	372	556	467

The following MCCL commands can be used to select among the motion trajectory profile modes:

aPT set axis a to trapezoidal profile

aPP set axis a to parabolic profile

aPS set axis a to s-curve (sinusoidal) profile

aPSn set axis a to s-curve (trapezoidal) profile
where n = ramp (0 < n < 0.5)

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